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Coupled Soft Fixed Point Theorems in Soft Metric and Soft b- Metric Space

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Abstract: In the present paper, we define Coupled Soft Metric Space. In the first part, we establish coupled soft fixed point theorem in soft metric space and in the second part of this paper, we prove coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with α -monotone property in an ordered soft b-metric space.

Keywords: Soft point, Soft metric space; Soft contractive mapping; α -monotone property.

1 Introduction and Preliminaries

The important part of fixed point theory is Metric fixed point theory, because of its applications in different areas like variational and linear inequalities, improvement and approximation theory. Ali et al. [14], Ahmadullah et al. [15], Agraval et al. ([16]-[18]) Pathak et al. [19] and many authors (see [9]-[13]) established fixed theorems in different spaces like Metric space, Manger space, Banach space etc. Wadkar et al. [12] proved coupled fixed point theorems in partially ordered metric space.

A concept of soft theory as new mathematical tool for dealing with uncertainties is discussed in 1999, by Molodtsov [6]. A collection of an approximate descriptions of an object is a soft set and this theory has rich potential applications. On soft set theory many structures contributed by many researchers (see [1], [4], [5], [8]). Shabir and Naz [7] were studied about soft topological spaces. In these studies, the concept of soft point is explained by different techniques. Recently Das and Samanta ([2, 3]) introduced concept of soft point, a notion of soft metric space and derived some basic properties of this spaces.

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In the present paper first we prove soft coupled fixed point theorem in soft metric space and then coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with α monotone property in an ordered soft b-metric space. Before going to prove our results first we define some important definitions.

Definition 1.1: Let X and E respectively be an initial inverse and a parameter sets. A soft set over X is pair denoted by (Y, E) iff Y is a mapping such that $Y : E \to P(x)$, where P(x) is the power set of X.
Definition 1.2: The intersection of two soft sets (Y,A) and (Z,B) over X is a soft set denoted by (I,C) over X and is given by $(Y,A)\tilde{\cap}(Z,B) = (I,C)$, where $C = A \cap B$ and $\forall \varepsilon \in C, I(\varepsilon) = Y(\varepsilon) \cap Z(\varepsilon)$.
Definition 1.3: The union of two soft sets (Y,A) and(Z,B) over X is the soft set (I,C), where $C = A \cup B$ and for all $k \in$ C,

$$I(k) = \begin{cases} Y(k), & \text{if k} \in \text{A} - \text{B}; \\ Z(k), & \text{if k} \in \text{B} - \text{A}; \\ Y(k) \cup Z(k), & \text{if k} \in \text{A} \cap \text{B}. \end{cases}$$

This is denoted by $(Y,A)\tilde{\cup}(Z,B) = (I,C)$.
Definition 1.4: For all k in A, if $Y(k) = \phi$ then soft set (Y,A) over X is called a null soft set denoted by ϕ.
Definition 1.5: For all k εA if $Y(k) = X$ then (Y,A) is called an absolute soft set over X.
Definition 1.6: The difference of two soft sets (F,E) and (G,E) over X is a soft set (H,E) over X and is denoted by $(F,E)\backslash(G,E)$ and is defined as $H(x) = F(x)\backslash G(x)$ for all x εE .
Definition 1.7: The complement soft set of (Y,A) is denoted by $(Y,A)^c$ and is defined as $(Y,A)^c = (Y^c,A)$, where $Y^c : A \to P(X)$ is a mapping given by $Y^c(\beta) = X - Y(\beta)$, for all β.
Definition 1.8: Let R be the set of real numbers and B(R) be the collection of all nonempty bounded subsets of R and E taken as a set of parameters. Then a mapping $Y : E \to B(R)$ is a soft real set and it is denoted by (Y,E).
Definition 1.9: For two soft real numbers $\tilde{m}$ and $\tilde{n}$ the following conditions hold:

1. $\tilde{m} \leq \tilde{n}$ if $\tilde{m}(s) \leq \tilde{n}(s)$,for all $s\varepsilon E$;
2. $\tilde{m} \geq \tilde{n}$ if $\tilde{m}(s) \geq \tilde{n}(s)$ for all $s\varepsilon E$;
3. $\tilde{m} < \tilde{n}$ if $\tilde{m}(s) < \tilde{n}(s)$ for all $s\varepsilon E$;
4. $\tilde{m} > \tilde{n}$ if $\tilde{m}(s) > \tilde{n}(s)$ for all $s\varepsilon E$.

Definition 1.10: A soft set (P,E) over X is said to have a soft point if there is exactly one $s\varepsilon E$ such that $P(s) = x$ for some $x\varepsilon X$ and $P(s') = \phi,\ foralls'\varepsilon E/\{s\}$. It will be denoted by $\tilde{x}_s$.

Definition 1.11: Two soft points $\tilde{x}_i$,$\tilde{y}_j$ are said to be equal if $i = j$ and $P(i) = P(j)$ i.e. $x = y$. Hence $\tilde{x}_i \neq \tilde{y}_j \Leftrightarrow x \neq y$ or $i \neq j$
Definition 1.12: A mapping $\tilde{\rho} : SP(\tilde{X}) \times SP(\tilde{X}) \to R(E)^*$ be soft mapping on $\tilde{X}$ such that:

SM1. for all $\tilde{x}_{s_1}, \tilde{y}_{s_2} \in \tilde{X}$, $\tilde{\rho}\left(\tilde{x}_{s_1}, \tilde{y}_{s_2}\right) \tilde{\geq} \bar{0}$;

SM2. $\tilde{\rho}\left(\tilde{x}_{s_1}, \tilde{y}_{s_2}\right) = \bar{0}$ if and only if $\tilde{x}_{s_1} = \tilde{y}_{s_2}$;

SM3. for all $\tilde{x}_{s_1}, \tilde{y}_{s_2} \in \tilde{X}, \tilde{\rho}\left(\tilde{x}_{s_1}, \tilde{y}_{s_2}\right) = \tilde{\rho}\left(\tilde{y}_{s_2}, \tilde{x}_{s_1}\right)$;

SM4. for all $\tilde{x}_{s_1}, \tilde{y}_{s_2}, \tilde{z}_{s_3} \in \tilde{X}, \tilde{\rho}\left(\tilde{x}_{s_1}, \tilde{z}_{s_3}\right) \leq \tilde{\rho}\left(\tilde{x}_{s_1}, \tilde{y}_{s_2}\right) + \tilde{\rho}\left(\tilde{y}_{s_2}, \tilde{z}_{s_3}\right)$.

Then the soft set $\tilde{X}$ with a soft metric $\tilde{\rho}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{\rho}, E)$.
Definition 1.13: Let us consider a soft metric $(\tilde{X}, \tilde{\rho}, E)$ and $\tilde{\alpha}$ be a non negative soft real number. The soft open ball with center at $\tilde{x}_s$ and radius $\tilde{\alpha}$ is given by

$$B(\tilde{x}_s, \tilde{\alpha}) = \left\{\tilde{y}_{s'} \in \tilde{X} : \tilde{\rho}(\tilde{x}_s, \tilde{y}_{s'}) \leq \tilde{\alpha}\right\} \subset SP(\tilde{X}).$$

and the soft closed ball with center at $\tilde{x}'_s$ and radius $\tilde{\alpha}$ is given by

$$B(\tilde{x}_s, \tilde{\alpha}) = \left\{\tilde{x}_{s'} \in \tilde{X} : \tilde{\rho}(\tilde{x}_s, \tilde{y}_{s'}) \leq \tilde{\alpha}\right\} \subset SP(\tilde{X}).$$

Definition1.14: A sequence $\left\{\tilde{x}^n_{\lambda_n}\right\}$ of soft points in soft metric space $(\tilde{X}, \tilde{\rho}, E)$ is said to be convergent in $(\tilde{X}, \tilde{\rho}, E)$, if there is a soft point $\tilde{y}_\mu \tilde{\in} \tilde{X}$ such that $\tilde{\rho}\left(\tilde{x}^n_{\lambda_n}, \tilde{y}_\mu\right) \to \tilde{0}$ as $n \to \infty$. That is for every $\tilde{\varepsilon} > \tilde{0}$, chosen arbitrary, there is a natural number $N = N(\tilde{\varepsilon})$ such that $\bar{0} \leq \tilde{\rho}(\tilde{x}^n_{\lambda_n}, \tilde{y}_\mu) \tilde{<} \tilde{\varepsilon}$, whenever $n > N$.
Definition1.15: Let $\left(\tilde{X}, \tilde{\rho}, E\right)$ be a soft metric space. A sequence $\left\{\tilde{x}^n_{\lambda_n}\right\}$ of soft points in soft metric space $(\tilde{X}, \tilde{\rho}, E)$ is said to be a Cauchy sequence in $\tilde{X}$, if corresponding to every $\tilde{\varepsilon} \tilde{\geq} \bar{0}$, there exist $m \in N$ such that $\tilde{\rho}\left(\tilde{x}^i_{\lambda_i}, \tilde{y}^j_{\lambda_j}\right) \tilde{\leq} \tilde{\varepsilon}, \forall i, j \geq m$, i.e. $\tilde{\rho}\left(\tilde{x}^i_{\lambda_i}, \tilde{y}^j_{\lambda_j}\right) \to \bar{0}$ as $i, j \to \infty$.
Definition1.16: The soft metric space $(\tilde{X}, \tilde{\rho}, E)$ is called complete, if every Cauchy sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.
Definition1.17: Let $(\tilde{X}, \tilde{\rho}, E)$ be a soft metric space. A function $(f, \varphi) : \left(\tilde{X}, \tilde{\rho}, E\right) \to \left(\tilde{X}, \tilde{\rho}, E\right)$ is called a soft contraction mapping if there exist, a soft real number $\alpha \varepsilon \tilde{R}, \tilde{0} \leq \alpha < \tilde{1}$ such that for every point $\tilde{x}_\lambda, \tilde{y}_\mu \varepsilon SP(X)$, we have

$$\tilde{\rho}\left((f, \varphi)(\tilde{x}_\lambda), (f, \varphi)(\tilde{y}_\mu)\right) \leq \alpha \tilde{\rho}\left(\tilde{x}_\lambda, \tilde{y}_\mu\right).$$

Definition 1.18: Let ($\tilde{X}, \leq$) be a partially ordered soft set. Let $S : X \times X \to X$ be a self map. One can say that S has the mixed monotone property if $S(x_\lambda, y_\mu)$ is monotone non-decreasing in x_λ and is monotone non-increasing in y_μ. That is for all $x^1_{\lambda 1}, x^2_{\lambda 2}$,

$$x^1_{\lambda_1} \leq x^2_{\lambda_2} \Rightarrow S\left(x^1_{\lambda_1}, y_\mu\right) \leq S\left(x^2_{\lambda_2}, y_\mu\right),$$

for any $y_\mu \in$ X and for all $y^1_{\mu_1}, y^2_{\mu_2}$,

$$y^1_{\mu_1} \geq y^2_{\mu_2} \Rightarrow S\left(x_\lambda, y^1_{\mu_1}\right) \geq S\left(x_\lambda, y^2_{\mu_2}\right),$$

for any $x_\lambda \in X$.

Definition 1.19: Consider partially ordered soft set. Let $(\tilde{X}, \leq)$ with $S : X \to X$ and $\alpha : X \to X$ be two mappings. We say that S has the mixed α -monotone property if S is monotone α non-decreasing in its first argument and is monotone α - non-increasing in its second argument, ie. for all $x^1_{\lambda 1}, x^2_{\lambda 2} \varepsilon X$,

$$\alpha x^1_{\lambda 1} \leq \alpha x^2_{\lambda 2} \Rightarrow S(x^1_{\lambda 1}, y_\mu) \leq S(x^2_{\lambda 2}, y_\mu),$$

for any $y_\mu \varepsilon X$ and for all, $y^1_{\mu 1}, y^2_{\mu 2} \varepsilon X$,

$$\alpha y^1_{\mu 1} \leq \alpha y^2_{\mu 2} \Rightarrow S(x_{\lambda 1}, y^1_\mu) \geq S(x_\lambda, y^2_{\mu 2}),$$

for any $x_\lambda \varepsilon X$.

Definition 1.20: An element $(x_\lambda, y_\mu) \varepsilon X \times X$ is said to be a coupled fixed point of mapping S: $X \to X$ if $S(x_\lambda, y_\mu) = x_\lambda$ and $S(y_\mu, x_\lambda) = y_\mu$.

Definition 1.21: An element is called $(x_\lambda, y_\mu) \in X \times X$ is called

1. a coupled coincidence soft point of mapping $S : X \to X$ and $\alpha : X \to X$ if $S(x_\lambda, y_\mu) = \alpha x_\lambda$ and $S(y_\mu, x_\lambda) = \alpha y_\mu$.
2. a common coupled soft fixed point of mapping $S : X \to X$ and $\alpha : X \to X$ if $x_\lambda = \alpha x_\lambda = S(x_\lambda, y_\mu) = \alpha x_\lambda$ and $y_\mu = \alpha y_\mu$= $S(y_\mu, x_\lambda)$.

Definition 1.22: For a non empty set X, the mappings $S : X \times X \to X$ and $\alpha : X \to X$ are said to be commutative, if for all $x_\lambda, y_\mu \varepsilon X$, we have

$$\alpha(S(x_\lambda, y_\mu)) = (S(\alpha x_\lambda, \alpha y_\mu)).$$

Definition 1.23: Let X be a non empty soft set and $s \geq 1$ be a given real number. Let $\tilde{\rho} : X \times X \to R^+$ be a function such that:

SBM1. for all $\tilde{x}_{s1}, \tilde{y}_{s2} \varepsilon \tilde{X}, \tilde{\rho}(\tilde{x}_{s1}, \tilde{y}_{s2}) \tilde{\geq} \bar{0}$;

SBM2. $\tilde{\rho}(\tilde{x}_{s1}\ ,\tilde{y}_{s2}) = \bar{0}$, if and only if $\tilde{x}_{s1} = \tilde{y}_{s2}$;

SBM3. for all $\tilde{x}_{s1}, \tilde{y}_{s2} \varepsilon \tilde{X}, \tilde{\rho}(\tilde{x}_{s1}, \tilde{\rho}\ (\tilde{x}_{s_1}, \tilde{y}_{s_2}) = \tilde{\rho}\ (\tilde{y}_{s_2}, \tilde{x}_{s_1})$;

SBM4. for all , $\tilde{x}_{s1}, \tilde{y}_{s2}, \tilde{z}_{s3} \varepsilon \tilde{X}\ , \tilde{\rho}(\tilde{x}_{s1}, \tilde{z}_{s3}) \leq s\{\tilde{\rho}(\tilde{x}_{s1}, \tilde{y}_{s2}) + \tilde{\rho}(\tilde{y}_{s2}, \tilde{z}_{s3})\}$.

Then soft set $\tilde{X}$ with a soft metric $\tilde{\rho}$ on $\tilde{X}$ is called a soft b-metric space and denoted by $(\tilde{X}, \tilde{\rho}, E)$

2 Main Results

Let $(X, \leq)$ be a partially ordered soft set and $\tilde{\rho}$ be a soft metric on X such that $(\tilde{X}, \tilde{\rho}, E)$ is a complete soft metric space.
Consider a product $(X, \tilde{\rho}, E) \times (X, \tilde{\rho}, E)$ with the following partial order.
For all $(x_\lambda, y_\mu), (u_\lambda, v_\mu) \varepsilon (X, \tilde{\rho}, E) \times (X, \tilde{\rho}, E)$, we have

$$(u_\lambda, v_\mu) \leq (x_\lambda, y_\mu) \Leftrightarrow x_\lambda \geq u_\lambda, y_\lambda \leq v_\lambda.$$

Theorem 2.1: Let $((X, \tilde{\rho}, E), \leq)$ be a partially ordered complete soft metric space. Let S be a continuous mapping having the mixed monotone property such that for a,b,c $\varepsilon\,[0,1)$ and for all $x_\lambda, y_\mu, u_\lambda, v_\mu$ in $(X, \tilde{\rho}, E), x_\lambda \neq y_\mu$, we have

$$\tilde{\rho}\left(S(x_\lambda, y_\mu), S(u_\lambda, v_\mu)\right) \leq a \max\left\{ \frac{\tilde{\rho}\left(x_\lambda, S(x_\lambda, y_\mu)\right)\tilde{\rho}\left(u_\lambda, S(u_\lambda, v_\mu)\right)}{\tilde{\rho}(x_\lambda, u_\lambda)}, \frac{\tilde{\rho}\left(u_\lambda, S(x_\lambda, y_\mu)\right)\tilde{\rho}\left(x_\lambda, S(u_\lambda, v_\mu)\right)}{\tilde{\rho}(x_\lambda, u_\lambda)} \right\}$$

$$+ b\tilde{\rho}\left(x_\lambda, u_\lambda\right) + c\left\{ \begin{array}{l} \tilde{\rho}\left(x_\lambda, S(x_\lambda, y_\mu)\right) + \tilde{\rho}\left(x_\lambda, S(u_\lambda, v_\mu)\right) \\ + \tilde{\rho}\left(u_\lambda, S(x_\lambda, y_\mu)\right) + \tilde{\rho}\left(u_\lambda, S(u_\lambda, v_\mu)\right) \end{array} \right\}, \tag{1}$$

where $a + b + 4c < 1$, then S has a coupled soft fixed point in $(X, \tilde{\rho}, E)$.

Proof: Choose $x^0_\lambda, y^0_\mu \varepsilon$ (X,$\tilde{\rho}, E) \times (X, \tilde{\rho}, E)$ and set

$$x^1_{\lambda 1} = S(x^0_\lambda, y^0_\mu)$$

and

$$y^1_{\mu 1} = S(y^0_\lambda, x^0_\lambda)$$

In general

$$x^{n+1}_{\lambda_{n+1}} = S(x^n_{\lambda_n}, y^n_{\mu_n})$$

and

$$y^{n+1}_{\mu n+1} = S(y^n_{\mu_n}, x^n_{\lambda_n})$$

with

$$x_\lambda^0 \leq S(x_\lambda^0, y_\mu^0) = x_{\lambda 1}^1$$

and

$$y_\mu^0 \geq S(y_\mu^0, x_\lambda^0) = y_{\mu 1}^1. \tag{2}$$

By iterative process above , we have $x_{\lambda 2}^2 = S(x_{\lambda 1}^1, y_{\mu 1}^1)$ and $y_{\mu 2}^2 = S(y_{\mu 1}^1, x_{\lambda 1}^1)$
Therefore

$$x_{\lambda 2}^2 = S(x_{\lambda 1}^1, y_{\mu 1}^1) = S(S(x_\lambda^0, y_\mu^0), S(y_\mu^0, x_\lambda^0)) = S^2(x_\lambda^0, y_\mu^0),$$

and

$$y_{\mu 2}^2 = S(y_{\mu 1}^1, x_{\lambda 1}^1) = S(S(y_\mu^0, x_\lambda^0), S(x_\lambda^0, y_\mu^0)) = S^2(y_\mu^0, x_\lambda^0).$$

Due to the mixed monotone property of S, we obtain

$$x_{\lambda 2}^2 = S^2(x_\lambda^0, y_\mu^0) = S(x_{\lambda 1}^1, y_{\mu 1}^1) \geq S(x_\lambda^0, y_\mu^0)) = x_{\lambda 1}^1,$$

and

$$y_{\mu 2}^2 = S^2(y_\mu^0, x_\lambda^0 = S(y_{\mu 1}^1, x_{\lambda 1}^1) \leq S(x_\lambda^0, y_\mu^0)) = y_{\mu 1}^1.$$

In general we have for $n \varepsilon N$

$$x_{\lambda_{n+1}}^{n+1} = S^{n+1}\left(x_\lambda^0, y_\mu^0\right) = S\left(S^n\left(x_\lambda^0, y_\mu^0\right), S^n\left(y_\mu^0, x_\lambda^0\right)\right),$$

and

$$y_{\mu_{n+1}}^{n+1} = S^{n+1}\left(y_\mu^0, x_\lambda^0\right) = S\left(S\left(y_\mu^0, x_\lambda^0\right), S\left(x_\lambda^0, y_\mu^0\right)\right). \tag{3}$$

It is obivious that

$$x_\lambda^0 \leq S\left(x_\lambda^0, y_\mu^0\right) = x_{\lambda_1}^1 \leq S^2\left(x_\lambda^0, y_\mu^0\right) = x_{\lambda_2}^2 \leq .... \leq S^n\left(x_\lambda^0, y_\mu^0\right) = x_{\lambda_n}^n \leq ....., \tag{4}$$

$$y_\mu^0 \geq S\left(y_\mu^0, x_\lambda^0\right) = y_{\mu_1}^1 \geq S\left(y_{\mu_1}^1, x_{\lambda_1}^1\right) = y_{\mu_2}^2 \geq ..... \geq S^n\left(x_\lambda^0, y_\mu^0\right) = y_{\lambda_n}^n \geq ....$$

Thus by mathematical induction principal, we have for $n \varepsilon N$

$$x_\lambda^0 \leq x_{\lambda_1}^1 \leq x_{\lambda_2}^2 \leq .... \leq x_{\lambda_n}^n \leq x_{\lambda_{n+1}}^{n+1} ....$$

$$y_\mu^0 \geq y_{\mu_1}^1 \geq y_{\mu_2}^2 \geq ..... \geq y_{\lambda_n}^n \geq y_{\lambda_{n+1}}^{n+1} ....$$

Thus we have by condition (1) that

$$\begin{aligned}
\tilde{\rho}\left(x^{n+1}_{\lambda_{n+1}}, x^{n}_{\lambda_n}\right) &= \tilde{\rho}\left(S\left(x^{n}_{\lambda_n}, y^{n}_{\mu_n}\right), S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right)\right) \\
&\leq a \max\left\{\begin{array}{c} \frac{\tilde{\rho}\left(x^{n}_{\lambda_n}, S\left(x^{n}_{\lambda_n}, y^{n}_{\mu_n}\right)\right)\tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right)\right)}{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right)}, \\ \frac{\tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, S\left(x^{n}_{\lambda_n}, y^{n}_{\mu_n}\right)\right)\tilde{\rho}\left(x^{n}_{\lambda_n}, S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right)\right)}{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right)} \end{array}\right\} \\
&\quad + b\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right) \\
&\quad + c\left\{\begin{array}{l} \tilde{\rho}\left(x^{n}_{\lambda_n}, S\left(x^{n}_{\lambda_n}, y^{n}_{\mu_n}\right)\right) + \tilde{\rho}\left(x^{n}_{\lambda_n}, S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right)\right) \\ + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, S\left(x^{n}_{\lambda_n}, y^{n}_{\mu_n}\right)\right) + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right)\right) \end{array}\right\}
\end{aligned}$$

$$\begin{aligned}
&\leq a \max\left\{\frac{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right)\tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n}_{\lambda_n}\right)}{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right)}, \frac{\tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n+1}_{\lambda_{n+1}}\right)\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n}_{\lambda_n}\right)}{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right)}\right\} \\
&\quad + b\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right) \\
&\quad + c\left\{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n}_{\lambda_n}\right) + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n}_{\lambda_n}\right)\right\} \\
&\leq a\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + b\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right) \\
&\quad + c\left\{\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n}_{\lambda_n}\right) + \tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(x^{n-1}_{\lambda_{n-1}}, x^{n}_{\lambda_n}\right)\right\}
\end{aligned}$$

Hence

$$\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) \leq (a+2c)\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + (b+2c)\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right).$$

Which implies that

$$\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) \leq \frac{b+2c}{1-a-2c}\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right). \tag{5}$$

Similarly, since $y^{n-1}_{\lambda_{n-1}} \geq y^{n}_{\lambda_n}$ and $x^{n-1}_{\lambda_{n-1}} \leq x^{n}_{\lambda_n}$, from (1) we have

$$\rho\left(y^{n}_{\lambda_n}, y^{n+1}_{\lambda_{n+1}}\right) \leq \frac{b+2c}{1-a-2c}\tilde{\rho}\left(y^{n}_{\lambda_n}, y^{n-1}_{\lambda_{n-1}}\right). \tag{6}$$

Adding (5) and (6),we get

$$\begin{aligned}
\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(y^{n}_{\lambda_n}, y^{n+1}_{\lambda_{n+1}}\right) &\leq \tfrac{b+2c}{1-a-2c}\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right) + \tfrac{b+2c}{1-a-2c}\tilde{\rho}\left(y^{n}_{\lambda_n}, y^{n-1}_{\lambda_{n-1}}\right) \\
&= \tfrac{b+2c}{1-a-2c}\left[\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n-1}_{\lambda_{n-1}}\right) + \tilde{\rho}\left(y^{n}_{\lambda_n}, y^{n-1}_{\lambda_{n-1}}\right)\right]
\end{aligned}$$

Let us denote $h = \frac{b+2c}{1-a-2c}$ and $\tilde{\rho}\left(x^{n}_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(y^{n}_{\lambda_n}, y^{n+1}_{\lambda_{n+1}}\right)$ by d_n then, $d_n \leq hd_{n-1}$.

Similarly it can be proved that $d_{n-1} \leq hd_{n-2}$.
Therefore

$$d_n \leq hd_{n-1} \leq h^2 d_{n-2} \leq \ldots\ldots.. \leq h^n d_0. \tag{7}$$

This implies that $\lim_{n\to\infty} d_n = 0$. Thus $\lim_{n\to\infty} d\left(x^{n+1}_{\lambda_{n+1}}, x^n_{\lambda_n}\right) = \lim_{n\to\infty} d\left(y^{n+1}_{\lambda_{n+1}}, y^n_{\lambda_n}\right) = 0$.
By equation (7), for each $m \geq n$ and repeat the application of triangle inequality, we obtain that

$$\tilde{\rho}\left(x^n_{\lambda_n}, x^m_{\lambda_m}\right) \leq \tilde{\rho}\left(x^n_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(x^{n+1}_{\lambda_{n+1}}, x^{n+2}_{\lambda_{n+2}},\right) + \tilde{\rho}\left(x^{n+2}_{\lambda_{n+2}}, x^{n+3}_{\lambda_{n+3}}\right) + \ldots\ldots.. + \tilde{\rho}\left(x^{m-1}_{\lambda_{m-1}}, x^m_{\lambda_m}\right),$$

and

$$\tilde{\rho}\left(y^n_{\lambda_n}, y^m_{\lambda_m}\right) \leq \tilde{\rho}\left(y^n_{\lambda_n}, y^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(y^{n+1}_{\lambda_{n+1}}, y^{n+2}_{\lambda_{n+2}},\right) + \tilde{\rho}\left(y^{n+2}_{\lambda_{n+2}}, y^{n+3}_{\lambda_{n+3}}\right) + \ldots\ldots.. + \tilde{\rho}\left(y^{m-1}_{\lambda_{m-1}}, y^m_{\lambda_m}\right).$$

Adding these we get

$$\begin{aligned}
\tilde{\rho}\left(x^n_{\lambda_n}, x^m_{\lambda_m}\right) + \tilde{\rho}\left(y^n_{\lambda_n}, y^m_{\lambda_m}\right) \leq\ & \tilde{\rho}\left(x^n_{\lambda_n}, x^{n+1}_{\lambda_{n+1}}\right) + \tilde{\rho}\left(y^n_{\lambda_n}, y^{n+1}_{\lambda_{n+1}}\right) \\
& + \tilde{\rho}\left(x^{n+1}_{\lambda_{n+1}}, x^{n+2}_{\lambda_{n+2}},\right) + \tilde{\rho}\left(y^{n+1}_{\lambda_{n+1}}, y^{n+2}_{\lambda_{n+2}},\right) \\
& + \tilde{\rho}\left(x^{n+2}_{\lambda_{n+2}}, x^{n+3}_{\lambda_{n+3}}\right) + \tilde{\rho}\left(y^{n+2}_{\lambda_{n+2}}, y^{n+3}_{\lambda_{n+3}}\right) + \ldots\ldots.. \\
& + \tilde{\rho}\left(x^{m-1}_{\lambda_{m-1}}, x^m_{\lambda_m}\right) + \tilde{\rho}\left(y^{m-1}_{\lambda_{m-1}}, y^m_{\lambda_m}\right) \\
\leq\ & [h^n + h^{n+1} + h^{n+2} + h^{n+3} + \ldots.. + h^{m-1}]d_0 \\
\leq\ & \tfrac{h^n}{1-h^n} d_0 \to 0, \text{ as } n \to \infty.
\end{aligned}$$

Therefore $\left\{x^n_{\lambda_n}\right\}$ and $\left\{y^n_{\mu_n}\right\}$ are Cauchy sequences. Since $(X, \tilde{\rho}, E)$ is complete soft metric space. There exists x_λ, y_μ in $(X, \tilde{\rho}, E)$ such that $\lim_{n\to\infty} x^n_{\lambda_n} = x_\lambda$ and $\lim_{n\to\infty} y^n_{\mu_n} = y_\mu$. Thus by taking limit $n \to \infty$, in equation (4) we get

$$x_\lambda = \lim_{n\to\infty} x^n_{\lambda_n} = \lim_{n\to\infty} S\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right) = S \lim_{n\to\infty}\left(x^{n-1}_{\lambda_{n-1}}, y^{n-1}_{\mu_{n-1}}\right) = S\left(x_\lambda, y_\mu\right),$$

and

$$y_\mu = \lim_{n\to\infty} y^n_{\mu_n} = \lim_{n\to\infty} S\left(y^{n-1}_{\mu_{n-1}}, x^{n-1}_{\lambda_{n-1}}\right) = S \lim_{n\to\infty}\left(y^{n-1}_{\mu_{n-1}}, x^{n-1}_{\lambda_{n-1}}\right) = S\left(y_\mu, x_\lambda\right).$$

Therefore $x_\lambda = S\left(x_\lambda, y_\mu\right)$and $y_\mu = S\left(y_\mu, x_\lambda\right)$. Thus S has coupled soft fixed point in $(X, \tilde{p}, E)$.

In the next theorem, we prove coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with α monotone property in an ordered soft b-metric space.

Theorem 2.2: Let $((X,\tilde{\rho},E),\leq)$ be a partially ordered set and $\tilde{\rho}:(X,\tilde{\rho},E)\times(X,\tilde{\rho},E)\rightarrow R$ be a soft-b metric defined on X with coefficient $s\geq 1$. Let $\alpha:(X,\tilde{\rho},E)\rightarrow(X,\tilde{\rho},E)$ and $S:(X,\tilde{\rho},E)\times(X,\tilde{\rho},E)\rightarrow(X,\tilde{\rho},E)$ be two mappings such that

$$\tilde{\rho}\left(S(x_\lambda,y_\mu),S(u_\lambda,v_\mu)\right)+\tilde{\rho}\left(S(y_\mu,x_\lambda),S(v_\mu,u_\lambda)\right)\leq k\left\{\tilde{\rho}(\alpha x_\lambda,\alpha u_\lambda)+\tilde{\rho}(\alpha y_\mu,\alpha v_\mu)\right\}, \tag{8}$$

for some $k\in\left[0,\frac{1}{s}\right)$ and for all $x_\lambda,y_\mu,u_\lambda,v_\mu\varepsilon(X,\tilde{\rho},E)$ with $ax_\lambda\geq au_\lambda$ and $ay_\mu\leq av_\mu$. We further assume the following hypothesis.

1. $S(X\times X)\subseteq\alpha(X)$;
2. $\alpha(x)$ is complete;
3. α is continuous and commute with S;
4. S has the mixed α - monotone property on X;
5. either S is continuous or X has the following property:

   a if a non decreasing sequence $\left\{x^n_{\lambda_n}\right\}\rightarrow x_\lambda$ then $\left\{x^n_{\lambda_n}\right\}\leq x_\lambda$;

   b if a non increasing sequence $\{y^n_{\mu_n}\}\rightarrow y_\mu$ then $\{y^n_{\mu_n}\}\leq y_\mu$.

If there exist two elements $x^0_{\lambda_0},y^0_{\mu_0}$ in $(X,\tilde{\rho},E)$ with $\alpha x^0_{\lambda_0}\leq S(x^0_{\lambda_0},y^0_{\mu_0})$ and $\alpha y^0_{\mu_0}\geq S(y^0,x^0_{\lambda_0})$, then S and α have unique a coupled soft fixed point. That is there exist a unique $x_\lambda\varepsilon(X,\tilde{\rho},E)$ such that $x_\lambda=S(x_\lambda,x_\lambda)=ax_\lambda$.

Proof: Let $x_{\lambda_0},y_{\mu_0}\varepsilon(X,\tilde{p},E)$ be such that $\alpha x_{\lambda_0}\leq S(x_{\lambda_0},y_{\mu_0})$ and $\alpha y_{\mu_0}\geq S(y_{\mu_0},x_{\lambda_0})$.
Since $S(X\times X)\geq\alpha(X)$, we can choose $x^1_{\lambda_1},y^1_{\mu_1}\varepsilon(X,\tilde{\rho},E)$ such that $\alpha x^1_{\lambda_1}=S(x_{\lambda_0},y_{\mu_0})$ and $\alpha y^1_{\mu_1}=S(y_{\mu_0},x_{\lambda_0})$. Again since $S(X\times X)\geq\alpha(X)$, we can choose $x^2_{\lambda_2},y^2_{\mu_2}\varepsilon(X,\tilde{\rho},E)$ such that $\alpha x^2_{\lambda_2}=S(x^1_{\lambda_1},y^1_{\mu_1})$ and $\alpha y^2_{\mu_2}=S(y^1_{\mu_1},x^1_{\lambda_1})$. Continuing this process we can construct two sequences $\{x^n_{\lambda_n}\}$ and $\{y^n_{\mu_n}\}$ in X such that

$$\alpha x^{n+1}_{\lambda_{n+1}}=S(x^n_{\lambda_n},y^n_{\mu_n}) \tag{9}$$

and

$$\alpha y^{n+1}_{\mu_{n+1}}=S(y^n_{\mu_n},x^n_{\lambda_n}),\forall n. \tag{10}$$

Now we will prove that for all $n\geq 0$.

$$\alpha x^n_{\lambda_n}\leq\alpha x^{n+1}_{\lambda_{n+1}} \tag{11}$$

$$\alpha y^n_{\mu_n}\geq\alpha y^{n+1}_{\mu_{n+1}} \tag{12}$$

We shall use the mathematical law of induction.
Let $n = 0$.
since $\alpha x_{\lambda_0} \leq S(x_{\lambda_0}, y_{\mu_0})$, $\alpha y_{\mu_0} \geq S(y_{\mu_0}, x_{\lambda_0})$ and $\alpha x^1_{\lambda_1} = S(x_{\lambda_0}, y_{\mu_0})$, $\alpha y^1_{\mu_1} = S(y_{\mu_0}, x_{\lambda_0})$, we have $\alpha x_{\lambda_0} \leq \alpha x^1_{\lambda_1}$ and $\alpha y_{\mu_0} \geq \alpha y^1_{\mu_1}$. That is (11) and (12) holds for all $n = 0$. We assume that (11) and (12) hold for some $n > 0$. As S has the mixed monotone property and $\alpha x^n_{\lambda_n} \leq \alpha x^{n+1}_{\lambda_{n+1}}$ and $\alpha y^n_{\mu_n} \geq \alpha y^{n+1}_{\mu_{n+1}}$, we get

$$\alpha x^{n+1}_{\lambda_{n+1}} = S(x^n_{\lambda_n}, y^n_{\mu_n}) \leq S(x^{n+1}_{\lambda_{n+1}}, y^n_{\mu_n}),$$

and

$$S(y^{n+1}_{\mu_{n+1}}, x^n_{\lambda_n}) \leq S(y^n_{\mu_n}, x^n_{\lambda_n}) = \alpha y^{n+1}_{\mu_{n+1}}.$$

Also for the same reason we have

$$\alpha x^{n+2}_{\lambda_{n+2}} = S(x^{n+1}_{\lambda_{n+1}}, y^{n+1}_{\mu_{n+1}}) \geq S(x^{n+1}_{\lambda_{n+1}}, y^n_{\mu_n})$$

and

$$S(y^{n+1}_{\mu_{n+1}}, x^n_{\lambda_n}) \leq S(y^n_{\mu_n}, x^n_{\lambda_n}) = \alpha y^{n+1}_{\mu_{n+1}}$$

From (9) and (10), we obtain

$$\alpha x^{n+1}_{\mu_{n+1}} \leq \alpha x^{n+2}_{\lambda_{n+2}}$$

and

$$\alpha y^{n+1}_{\mu_{n+1}} \geq \alpha y^{n+2}_{\mu_{n+2}}$$

Thus by mathematical induction we conclude that (11) and (12) holds for all $n \geq 0$.
Continuing this process, one can easily verify that

$$\alpha x_{\lambda_0} \leq \alpha x^1_{\lambda_1} \leq \alpha x^2_{\lambda_2} \leq .... \leq \alpha x^n_{\lambda_n} \leq \alpha x^{n+1}_{\lambda_{n+1}} \leq ... \tag{13}$$

$$y_{\mu_0} \geq \alpha y^1_{\mu_1} \geq \alpha y^2_{\mu_2} \geq ..... \geq \alpha y^n_{\lambda_n} \geq \alpha y^{n+1}_{\lambda_{n+1}}. \geq ... \tag{14}$$

Now if $(x^{n+1}_{\lambda_{n+1}}, y^{n+1}_{\mu_{n+1}}) = (x^n_{\lambda_n}, y^n_{\mu_n})$, then S and α have coupled soft coincidence point.
So assume $(x^{n+1}_{\lambda_{n+1}}, y^{n+1}_{\mu_{n+1}}) \neq (x^n_{\lambda_n}, y^n_{\mu_n})$ for all $n \geq 0$, i.e. we assume that either

$$\alpha x^{n+1}_{\lambda_{n+1}} = S(x^n_{\lambda_n} y^n_{\mu_n}) \neq \alpha x^n_{\lambda_n}$$

or

$$\alpha y^{n+1}_{\mu_{n+1}} = S(y^n_{\mu_n}, x^n_{\lambda_n}) \neq \alpha y^n_{\mu_n}.$$

Again

$$\begin{aligned}\tilde{\rho}\left(\alpha x^{n}_{\lambda_n},\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\alpha y^{n}_{\mu_n},\alpha y^{n+1}_{\mu_{n+1}}\right)&=\tilde{\rho}\left(S\left(x^{n-1}_{\lambda_{n-1}},y^{n11}_{\mu_{n11}}\right),S\left(x^{n}_{\lambda_n},y^{n}_{\mu_n}\right)\right)\\&+\tilde{\rho}\left(S\left(y^{n-1}_{\mu_{n-1}},x^{n-1}_{\lambda_{n-1}}\right),S\left(y^{n}_{\mu_n},x^{n}_{\lambda_n}\right)\right)\\&\leq k\left\{\tilde{\rho}\left(\alpha x^{n-1}_{\lambda_{n-1}},\alpha x^{n}_{\lambda_n}\right)+\tilde{\rho}\left(\alpha y^{n-1}_{\mu_{n-1}},\alpha y^{n}_{\mu_n}\right)\right\}.\end{aligned}$$

Now let

$$\tilde{\rho}\left(\alpha x^{n}_{\lambda_n},\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\alpha y^{n}_{\mu_n},\alpha y^{n+1}_{\mu_{n+1}}\right)=d_n,$$

then

$$d_n\leq kd_{n-1}. \tag{15}$$

which implies

$$d_n\leq kd_{n-1}\leq k^2d_{n-2}\leq k^3d_{n-3}\leq k^4d_{n-4}\leq k^5d_{n-5}......\leq k^nd_0.$$

Again let m and n be two positive integer such that $m>n$, then we can write

$$\begin{aligned}\tilde{\rho}\left(\alpha x^{n}_{\lambda_n},\alpha x^{m}_{\lambda_m}\right)&\leq s\left\{\tilde{\rho}\left(\alpha x^{n}_{\lambda_n},\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\alpha x^{n+1}_{\lambda_{n+1}},\alpha x^{m}_{\lambda_m}\right)\right\}\\&\leq s\tilde{\rho}\left(\alpha x^{n}_{\lambda_n},\alpha x^{n+1}_{\lambda_{n+1}}\right)+s^2\tilde{\rho}\left(\alpha x^{n+1}_{\lambda_{n+1}},\alpha x^{n+2}_{\lambda_{n+2}}\right)\\&+s^3\tilde{\rho}\left(\alpha x^{n+2}_{\lambda_{n+2}},\alpha x^{n+3}_{\lambda_{n+3}}\right)+........\\&+s^{m-n-1}\tilde{\rho}\left(\alpha x^{m-1}_{\lambda_{m-1}},\alpha x^{m}_{\lambda_m}\right).\end{aligned}$$

Similarly

$$\begin{aligned}\tilde{\rho}\left(\alpha y^{n}_{\mu_n},\alpha y^{m}_{\mu_m}\right)&\leq s\tilde{\rho}\left(\alpha y^{n}_{\mu_n},\alpha y^{n+1}_{\mu_{n+1}}\right)+s^2\tilde{\rho}\left(\alpha y^{n+1}_{\mu_{n+1}},\alpha y^{n+2}_{\mu_{n+2}}\right)\\&+s^3\tilde{\rho}\left(\alpha y^{n+2}_{\mu_{n+2}},\alpha y^{n+3}_{\mu_{n+3}}\right)+........\\&+s^{m-n-1}\tilde{\rho}\left(\alpha y^{m-1}_{\mu_{m-1}},\alpha y^{m}_{\mu_m}\right).\end{aligned}$$

Therefore

$$\tilde{\rho}\left(\alpha x^n_{\lambda_n}, \alpha x^m_{\lambda_m}\right) + \tilde{\rho}\left(\alpha y^n_{\mu_n}, \alpha y^m_{\mu_m}\right) \leq s d_n + s^2 d_{n+1} + s^3 d_{n+2} + \ldots\ldots.. + s^{m-n-1} d_{m-1}$$

$$\leq s k^n d_0 + s^2 k^{n+1} d_0 + s^3 k^{n+2} d_0 + \ldots\ldots.. + s^{m-n-1} k^n d_0$$

$$\leq s k^n d_0 \left[1 + sk + s^2 k^2 + s^3 k^3 + \ldots + s^{m0n-1} k^{m-n-1}\right]$$

$$\leq s k^n d_0 [\tfrac{1}{1-sk}]$$

$$\to 0, \mathrm{n} \to \infty.$$

Hence $\{x^n_{\lambda_n}\}$ and $\{y^n_{\mu_n}\}$ are two Cauchy sequences in X and X is complete. Thus there exist two soft point say x_λ, y_μ in X such that $\left\{\alpha x^n_{\lambda_n}\right\} = \alpha x_\lambda = \xi$ and $\{\alpha y^n_{\mu_n}\} \to \alpha y_\mu = \eta$ as $n \to \infty$. Hence S is complete and so

$$\alpha\left(\alpha x^{n+1}_{\lambda_{n+1}}\right) = \alpha\left(S\left(x^n_{\lambda_n}, y^n_{\mu_n}\right)\right) = S\left(\alpha x^n_{\lambda_n}, \alpha y^n_{\mu_n}\right),$$

($\because$ S and α are commutative)

$$\alpha(\xi) = S(\xi, \eta).$$

($\because$ S and α are continuous)

Similarly we can show that $\alpha(\eta) = S(\eta, \xi)$. Thus $(\eta\xi)$ is point of coincidence for S and α. Again let 5b holds, by (13) we get that $\{\alpha x^n_{\lambda_n}\}$ is a non decreasing sequence and $\alpha x^n_{\lambda_n} \to \xi$, therefore for all n. Similarly by (14), we get $\left\{\alpha y^n_{\mu_n}\right\}$ is a non increasing sequence and $\alpha x^n_{\lambda_n} \to \xi$, therefore $\alpha y^n_{\mu_n} \geq \eta$, for all n. Then

$$\begin{aligned}
\tilde{\rho}\left(\alpha(\xi), S(\xi,\eta)\right) &\leq s\tilde{\rho}\left(\alpha(\xi), \alpha\alpha x^{n+1}_{\lambda_{n+1}}\right) + s\tilde{\rho}\left(\alpha\alpha x^{n+1}_{\lambda_{n+1}}, S(\xi,\eta)\right) \\
&= s\tilde{\rho}\left(\alpha(\xi), \alpha\alpha x^{n+1}_{\lambda_{n+1}}\right) + s\tilde{\rho}\left(\alpha\left(S(x^n_{\lambda_n}, y^n_{\mu_n})\right), S(\xi,\eta)\right) \\
&= s\tilde{\rho}\left(\alpha(\xi), \alpha\alpha x^{n+1}_{\lambda_{n+1}}\right) + s\tilde{\rho}\left(S\left(\alpha x^n_{\lambda_n}, \alpha y^n_{\mu_n}\right)\right), S(\xi,\eta)\right) \\
&\leq \tilde{\rho}\left(\alpha(\xi), \alpha\alpha x^{n+1}_{\lambda_{n+1}}\right) + s\tilde{\rho}\left(S\left(\alpha x^n_{\lambda_n}, \alpha y^n_{\mu_n}\right)\right), S(\xi,\eta)\right) \\
&\quad + s\tilde{\rho}\left(S\left(\alpha y^n_{\mu_n}, \alpha x^n_{\lambda_n}\right)\right), S(\eta,\xi)\right).
\end{aligned}$$

Hence

$$\tilde{\rho}\left(\alpha(\xi), S(\xi,\eta)\right) \leq \tilde{\rho}\left(\alpha(\xi), \alpha\alpha x^{n+1}_{\lambda_{n+1}}\right) + sk\left\{\tilde{\rho}\left(\alpha\alpha x^n_{\lambda_n}, \alpha\xi\right) + s\tilde{\rho}\left(\alpha\alpha y^n_{\mu_n}, \alpha\eta\right)\right\} \quad (16)$$

Since α is continuous, $\alpha a x^n_{\lambda_n} \to a\xi$ and $\alpha a y^n_{\mu_n} \to a\eta$ and hence the right hand side of equation (16) becomes zero as $n \to \infty$. Thus $a(\xi) = S(\xi,\eta)$. Similarly we can show that $a(\eta) = S(\xi,\eta)$. Again

$$\begin{aligned}\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)+\tilde{\rho}\left(\alpha\eta,\alpha\xi\right) &= \tilde{\rho}\left(S(\xi,\eta),S(\eta,\xi)\right)+\tilde{\rho}\left(S(\eta,\xi),S(\xi,\eta)\right)\\ &\leq k\left\{\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)+\tilde{\rho}\left(\alpha\eta,\alpha\xi\right)\right\}.\end{aligned}$$

This implies

$$2\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)\leq 2k\tilde{\rho}\left(\alpha\xi,\alpha\eta\right) \ .$$

That is $\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)\leq k\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)$. Since $k=\frac{1}{s}$,$\tilde{\rho}\left(\alpha\xi,\alpha\eta\right)=0$. Thus $\alpha\xi=\alpha\eta$.
Hence $S\left(\xi,\eta\right)=\alpha\left(\xi\right)=\alpha(\eta)=S\left(\eta,\xi\right)$.
Finally,

$$\tilde{\rho}\left(\xi,\alpha\xi\right)\leq s\tilde{\rho}\left(\xi,\alpha x^{n+1}_{\lambda_{n+1}}\right)+s\tilde{\rho}\left(\alpha x^{n+1}_{\lambda_{n+1}},\alpha\xi\right)\leq s\tilde{\rho}\left(\xi,\alpha x^{n+1}_{\lambda_{n+1}}\right)+s\tilde{\rho}\left(S(x^n_{\lambda_n},y^n_{\mu_n}),S(\xi,\eta)\right),$$

and in the same manner

$$\tilde{\rho}\left(\eta,\alpha\eta\right)\leq s\tilde{\rho}\left(\eta,\alpha y^{n+1}_{\mu_{n+1}}\right)+s\tilde{\rho}\left(\alpha y^{n+1}_{\mu_{n+1}},\alpha\eta\right)\leq s\tilde{\rho}\left(\eta,\alpha y^{n+1}_{\mu_{n+1}}\right)+s\tilde{\rho}\left(S(y^n_{\mu_n},x^n_{\lambda_n}),S(\eta,\xi)\right).$$

Therefore

$$\begin{aligned}\tilde{\rho}\left(\xi,\alpha\xi\right)+\tilde{\rho}\left(\eta,\alpha\eta\right) &\leq s\left\{\tilde{\rho}\left(\xi,\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\eta,\alpha y^{n+1}_{\mu_{n+1}}\right)\right\}+\\ &+s\left[\tilde{\rho}\left(S(x^n_{\lambda_n},y^n_{\mu_n}),S(\xi,\eta)\right)+\tilde{\rho}\left(S(y^n_{\mu_n},x^n_{\lambda_n}),S(\eta,\xi)\right)\right]\\ &\leq s\left\{\tilde{\rho}\left(\xi,\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\eta,\alpha y^{n+1}_{\mu_{n+1}}\right)\right\}\\ &+s^2\left[\tilde{\rho}\left(\alpha x^n_{\lambda_n},\xi\right)+\tilde{\rho}\left(\xi,\alpha\xi\right)+\tilde{\rho}\left(\alpha y^n_{\mu_n},\eta\right)+\tilde{\rho}\left(\eta,\alpha\eta\right)\right].\end{aligned}$$

Thus

$$\begin{aligned}\left(1-ks^2\right)\left[\tilde{\rho}\left(\xi,\alpha\xi\right)+\tilde{\rho}\left(\eta,\alpha\eta\right)\right] &\leq s\left\{\tilde{\rho}\left(\xi,\alpha x^{n+1}_{\lambda_{n+1}}\right)+\tilde{\rho}\left(\eta,\alpha y^{n+1}_{\mu_{n+1}}\right)\right\}\\ &+s^2\left[\tilde{\rho}\left(\alpha x^n_{\lambda_n},\xi\right)+\tilde{\rho}\left(\alpha y^n_{\mu_n},\eta\right)\right]\to 0, \text{ as } n\to\infty.\end{aligned}$$

Hence $\tilde{\rho}\left(\xi,\alpha\xi\right))=0=\tilde{\rho}\left(\eta,\alpha\eta\right))$.
This implies that $\xi=\alpha\xi$ and $\eta=\alpha\eta$. That is $\alpha x_\lambda = S\left(x_\lambda,x_\lambda\right)=x_\lambda$. This means that S and α have a common soft fixed point.

3 Conclusion

In this paper the investigations concerning the existence and uniqueness of soft coupled fixed point mapping in soft metric space and soft b- metric space are established.

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