Applied Element Method (AEM) in Dynamic and Seismic Analysis
Earthquake Engineering and Engineering Seismology

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   - Numerical methods - FEM/AEM
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   - Extreme Loading® for Structures
Applied Element Method

Numerical methods

- Applied Element Method (AEM) - relatively new numerical method (15-20 years)
- Two general approaches in numerical methods:
  - Continuum based numerical methods (FEM, BEM, ...)
  - Discrete formulations (Rigid Body and Spring Models, Discrete Element Method, EDEM, ...)
- AEM evolved from Extended DEM and RBSM as the new discrete numerical method
Applied Element Method

Numerical methods

- 1\textsuperscript{st} comprehensive formulation:
  

- 1\textsuperscript{st} use of the name AEM:

Applied Element Method

Numerical methods

- Finite Element Method (FEM) - still the most widely used numerical method
- FEM simulations in all engineering fields (solid, fluid, acoustic, thermal, electro-magnetic, chemical engineering, ...)
- FEM applied in non-engineering fields too (medicine, ...)
- FEM applied in various interdisciplinary areas (fluid-structure, soil-structure, etc.)
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Connectivity between elements

- In the FEM elements are connected at nodes
- All deformation is happening inside the element itself
- In the AEM elements are connected through the whole surface using springs
- Each spring is actually 3 springs: 1 spring for normal and 2 springs for shear deformations
- In the AEM deformation is outside elements (in surface springs)
- Elements in AEM are rigid bodies
Various Finite Elements

Figure 1.1 Types of finite elements
Applied Elements

Figure 1.2 Applied element
FEM and AEM - Comparison

Element Connectivity & Auto-Element Separation:

FEM

- Full nodal compatibility
- Deformations inside elements

AEM

- Deformations outside elements
- Deformations in surface springs
Transition from large elements to small elements

- Due to various reasons (stress concentration, ...), areas of finer mesh are necessary
- Coarse and fine mesh in FEM: special transition elements
- In the AEM transition elements are not necessary
- Connection between elements in AEM is realized through the surface springs, not nodal contact
FEM and AEM - Comparison

FEM

AEM

Transition elements

No transition elements needed
Partial element connectivity

- FEM - connectivity and continuity at nodes: $C_0$, $C_1$
- In the FEM partial connectivity may be achieved only with additional elements and new dofs
- In the AEM partial connectivity is easily achieved with surface springs
- In the AEM discontinuity between elements may be easily achieved (zero spring stiffness)
FEM and AEM - Comparison

Comparison: FEM - AEM

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Numerical methods

- Problems with FEM in fields with discontinuities in material
- Simulations of crack initiation, development, propagation
- Special techniques in FEM for crack analysis (smeared crack approach, discrete crack approach,...)
- FEM simulation of conditions that define initiation of structural collapse - YES
- However, progressive structural collapse with FEM - NO
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Numerical methods

- AEM is the only numerical method which can accurately analyze, track and visualize structural behavior through all 3 stages of the loading:
  - Small displacements (linear elastic, non-linear)
  - Large displacements (geometric & material change, Element separation)
  - Collision and collapse (including progressive collapse)
- The only software related to AEM: Extreme Loading® for Structures (ELS)
Linear elastic analysis
Nonlinear analysis
Separation of elements
Collision and progressive collapse
FEM and AEM - Comparison

Table 3. Analysis Domains of Applied Element Method (Tagel-Din and Rahman 2006)

<table>
<thead>
<tr>
<th></th>
<th>Small Displacement</th>
<th>Large Displacement</th>
<th>Collision</th>
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<tbody>
<tr>
<td></td>
<td>Elastic</td>
<td>Geometrical and Material Changes</td>
<td>Element Separation</td>
</tr>
<tr>
<td>Small</td>
<td>Accurate</td>
<td>Reliable Results</td>
<td>Develop.</td>
</tr>
<tr>
<td>Displacement</td>
<td>Accurate</td>
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FEM and AEM - Comparison

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<td>Nonlinear</td>
<td></td>
<td>Progressive Collapse</td>
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</tbody>
</table>

**AEM** (Applied Element Method)

**FEM** (Finite Element Method)

* AEM is a completely new methodology for structural behavioral analysis during Progressive Collapse under Extreme Loads.

** FEM is the current industry design methodology used for static and non-linear analysis of structures.

*** Both Methodologies are very different, both architecturally and mathematically.
Introduction
Overview of the AEM
Dynamic analysis with the AEM
Springs in the AEM
Discretization in the AEM
Large deformation static analysis with AEM

Applied Element Method

Overview of the AEM

- Discretization of a structure into nodes, surrounded by a small volume elements (rectangular in 2D, parallelopiped in 3D)
- Each node in AEM has 3 or 6 dofs (2D or 3D)
- Elements are connected by normal and shear springs
- Springs are responsible for internal deformations and stress field in the structure
- Discrete elements around the nodal points move as the rigid bodies, but the element assembly is deformable
- Structural deformation is represented by the spring deformation
Degrees of freedom in the AEM

![Diagram showing degrees of freedom](image)

**Normal**
- Translation
- Rotation

**Shear x-z**
- Translation

**Shear x-y**
- Rotation

**Shear x-y**
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There are **3 types of springs** in the AEM:
- Matrix (material) springs
- Reinforcement springs
- Contact springs

Linear and/or non-linear (normal and shear) springs are representing the material behavior.

For the RC structures **each reinforcement bar** is represented by the corresponding normal and shear springs.

Contact springs are generated when elements collide with each other or with ground.
Material springs in the AEM
Reinforcement springs in the AEM

Normal Springs

Shear Springs x-z

Shear Springs y-z
Applied Element Method

Automatic element contact and collision in AEM

- Contact or collision in AEM is detected automatically
- Elements in AEM are able to contact and separate, re-contact again without user intervention
- There are 3 types of contacts in the AEM:
  - Corner-to-Face
  - Edge-to-Edge
  - Corner-to-Ground
Contact Corner-to-Face in the AEM

Corner to Face Contact:

Shear Spring in Y

Shear Spring in X

Normal Spring
Contact Edge-to-Edge in the AEM
Contact Corner-to-Ground in the AEM

Corner to Ground Contact:

Ground
Shear spring in y

Ground
Shear spring in X

Ground
Normal Spring
Collision between elements in the AEM

Element Collision:
Applied Element Method

Automatic element separation in the AEM

- When the average strain at the element face reaches the separation strain, all springs at that face are removed.
- Separation strain is the average strain along the element face corresponding to material properties.
- After removal of springs adjacent elements are no longer connected, until collision occurs.
- If the collision occurs, elements collide as the rigid bodies.
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Discretization in the AEM

- The structure is virtually divided into small elements (rectangular, 2D, or cubic, 3D)
- Elements have one nodal point in their centers
- Springs on element faces are connection between elements
- Each reinforcement is presented by the corresponding spring (each spring is, actually, 2 or 3 springs: normal and shear)
At the location of reinforcement two pairs of springs are used: for concrete and for reinforcement bar.

Reinforcement spring and concrete spring have the same strain.

Due to stress conditions, separation between elements occurs because of failure of a concrete-spring before the reinforcement-spring.

Therefore, relative displacement between reinforcement and concrete may be taken into account automatically.
Discretization in the AEM

(a) Element generation in AEM

(b) Spring distribution and area of influence
With reference to previous figure, spring stiffness are:

- Normal spring
  \[ K_n = \frac{E T d}{a} \]

- Shear spring
  \[ K_s = \frac{G T d}{a} \]

\( E \) and \( G \) are the modulus of elasticity and the shear modulus (of the material)
Spring stiffness in the AEM

- $T$ is the thickness of the element, $a$ the distance between element centroids (nodal points), i.e. the length of the spring, while $d$ is the distance between adjacent springs.

- Therefore, $Td$ is the affected area of the spring, while $a$ is the length of the spring.

- For the reinforcement spring, $Td$ is replaced by the area of the reinforcement bar $Ta \rightarrow A_a$. 
Two elements in an arbitrary position
Stiffness matrix in the AEM

- Stiffness matrix components corresponding to each dof are determined by assuming unit displacement in considered dof and by determining the corresponding force in the centroid of each element.

- For two elements in 2D, and dofs \( u_1, u_2, \ldots, u_6 \), the stiffness matrix is \( 6 \times 6 \):

\[
K = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\]

where submatrices \( K_{12} \) and \( K_{21} \) are symmetric: \( K_{12} = K_{21} \)
Stiffness matrix in the AEM

- Elements of the submatrix $K_{11}$ are given by $k_{ij} = k_{ji}$, $(i, j = 1, 2, 3)$

- Diagonal elements

$$k_{11} = \sin^2(\Theta + \alpha) K_n + \cos^2(\Theta + \alpha) K_s$$
$$k_{22} = \sin^2(\Theta + \alpha) K_s + \cos^2(\Theta + \alpha) K_n$$
$$k_{33} = L^2 \cos^2 \alpha K_n + L^2 \sin^2 \alpha K_s$$
Stiffness matrix in the AEM

- Off-diagonal elements

\[ k_{12} = -L \sin(\Theta + \alpha) \sin(\Theta + \alpha) K_n \]
\[ + L \sin(\Theta + \alpha) \cos(\Theta + \alpha) K_s \]
\[ k_{13} = L \cos(\Theta + \alpha) \sin \alpha K_s - L \sin(\Theta + \alpha) \cos \alpha K_n \]
\[ k_{23} = L \cos(\Theta + \alpha) \cos \alpha K_n + L \sin(\Theta + \alpha) \sin \alpha K_s \]

where \( L, \Theta, \alpha \) are given in the previous figure, while \( K_n \) and \( K_s \) are the normal and shear spring stiffness.
Stiffness matrix in the AEM

- Presented stiffness coefficients are due to only one pair of springs (normal and shear).
- The stiffness coefficients depend on the spring stiffness and the spring location.
- The global stiffness matrix is obtained by summing up the stiffness matrices of all pairs of springs around each element.
- The order of the global stiffness matrix is $3N$ or $6N$, where $N$ is the number of elements (i.e. the nodal points).
Linear static analysis in the AEM

- The governing equation in the linear elastic static analysis is

\[ K \mathbf{u} = \mathbf{f} \]

- Matrix \( K \) is the global stiffness matrix, \( \mathbf{u} \) is the unknown displacement vector and \( \mathbf{f} \) is the applied load vector
- Small displacement theory
- Obtained results (for sufficient number of elements and springs) are equal to the theoretical results
Number of elements and springs in the AEM

- Various parametric analyses in order to determine the number of elements and the number of springs over the face of each element.
- For framed structures the optimum is:
  - ≈ 10 elements along the cross-sectional height
  - ≈ 10 springs over each element face
- More elements and more springs give better accuracy, but also much greater CPU time.
Parametric analysis of the cantilever beam

Figure 3: Dimensions and element arrangements of laterally loaded cantilever models
Parametric analysis of the cantilever beam

Figure 4: Relations between the number of base elements, ratio of error and CPU time
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Large deformation static analysis in the AEM

- Large deformation static analysis: geometric and material non-linearity
- The governing equation for large deformation analysis:

\[ K \Delta u = \Delta f + R_m + R_g \]

- \( K \) is the nonlinear stiffness matrix, \( \Delta u \) incremental displacement vector, \( \Delta f \) incremental load vector
- \( R_m \) is the residual force vector due to cracking or incompatibility between spring strains and stresses
- \( R_g \) is the residual force vector due to geometric changes in the structure due to loading
Postbuckling behavior of the cantilever beam

Fig. 6  Post buckling behavior of an elastic column
Load-displacement relations of the cantilever beam

Figure 14: Load-displacement relation of an elastic cantilever under vertical load
Postbuckling behavior of the sway frame

Fig. 11  Load displacement relationship for a sway frame with fixed supports subjected to vertical loads
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Dynamic linear/nonlinear analysis in the AEM

- The governing equation for large deformation dynamic analysis:

\[ M \Delta \ddot{u} + C \Delta \dot{u} + K \Delta u = \Delta f(t) + R_m + R_g \]

- \( M \) are \( C \) are the mass and damping matrices and \( K \) is the nonlinear stiffness matrix
- \( \Delta \ddot{u}, \Delta \dot{u} \) and \( \Delta u \) are incremental acceleration, velocity and displacement vectors
- \( \Delta f(t) \) is the incremental load vector
Dynamic linear/nonlinear analysis in the AEM

- $R_m$ is the residual force vector due to cracking or incompatibility between spring strains and stresses

- $R_g$ is the residual force vector due to geometric changes in the structure due to loading
Eigenvalue analysis of a building

Figure 19: Eigen values and modes of the building model.
Introduction

Overview of the AEM

Dynamic analysis with the AEM

Differential equations of motion

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Seismic analysis of a building

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Applied Element Method
Seismic analysis of a building

Figure 4.1. AEM models of BF1 frame and corresponding seismic loading scenarios
Seismic analysis of a building: model

Model on the **shaking table** and numerical model
Seismic analysis of a building: model

Figure 17: Shape, dimensions and loading of a small-scaled RC building model under lateral excitation
Seismic analysis of a building: results

Graph showing displacement over time for Case 3 (v=0.02, dt=0.0025)
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Progresive collapse of a building

0.00 sec  2.47 sec  3.34 sec  4.09 sec

4.84 sec  5.59 sec

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Progresive collapse of a building


(a) Illustration of the 3-D model used in the nonlinear dynamic analysis of the studied structures using ELS.

(b) Zoom-in of part of the model showing its different components.
AEM - Demolition of a building

Figure 2.3 Real and simulated structure before demolition: a) real construction after preparatory works; b) geometrical model of structure; c) geometrical model of structure before starting simulation.
AEM - Demolition of a building

Figure 2.7 Numerical simulation and properly demolition
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Reinforcement presentation in ELS

Custom Reinforcement
Reinforcement yield in ELS

Automatic Yielding of Reinforcement

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Automatic crack propagation in ELS

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Collapse of a building in ELS

Automatic Separation

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